Clock-Dedekind cuts-Time

-Towards a Fractal Construction of Time-

(Formulation of the Problem)

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Abstract: This article approaches its subject in a philosophical manner. It has as objective to formulate a problem. The formulation of the problem is based on the Dedekind cuts and on a continuous function nowhere differentiable (it has no tangent line to the points of its graph): the Takagi-Knopp function. The study mainly approaches two contents: one from the algebra and one from the mathematical analysis. The interpretation of the formulation requires an epistemological frame. This study is only a first step. The next part of the study is the construction of another continuous function nowhere differentiable, for time. The main intention is to construct time only from "moments" understanding them as points of the graph in which the function is not differentiable.

Keywords: time, clock, measurement of time, Dedekind cut, continuous function nowhere differentiable, moment, points of the graph in which the function is not differentiable, fractal.

"Mathematical propositions express no thoughts.

In life it is never a mathematical proposition which we need, but we use mathematical propositions only in order to infer from propositions which do not belong to mathematics to others which equally do not belong to mathematics."[1]

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[1] Wittgenstein Ludwig - *Tractatus Logico-Philosophicus*, Ed. Humanitas, Bucureşti 2001, P 6.21; P 6.211.

For an epistemological analysis of the consistency and limits implied by a mathematical construction burdened with physical significance, this presentation has chosen, on the one hand, the construction of real numbers by Dedekind cuts for the operation of physical measurements. On the other hand, it has chosen a simple mathematical special function. This function meets two essential requirements: it is a continuous function and it is nowhere differentiable. The function meets the first condition as a mathematical requirement imposed by the most important attribute of physical time: continuity. The second condition is the mathematical expression of the intention to construct time only from "moments" understood as points of the graph in which the function is not differentiable (it has no tangent line to the points of its graph). The chosen function is just a first support of critical analysis of the problems which appear through such a construction. For the sake of clarity of both the exposition and the analysis, the presentation is made up of certain stages which try to intuitively suggest that which at the end is abstract and unintuitive. The present paper presents the first two stages.

In the graphical representation of the formulation of the problem, and for the sake of simplicity and suggestiveness of the exposition the axis Ot(🕐) will be called "clock". The graph of function K(t(🕐) – henceforth noted K(t) – or the mathematical curve Θ will be called TIME. We shall assume, in relation with the effective physical measurements that can be made by a clock, that the clock only indicates rational numbers, in other words, the dial of the clock corresponds to the rational numbers set $\mathbf{Q}$. We shall also assume that the physical measurement with a clock has its limits in principle, "that what a clock cannot measure", and Dedekind cuts correspond to these. However, in "reality", even in the absence of measurements, Dedekind cuts correspond to moments of time. Time Θ is in this context the mathematical image of all possible indications of a clock together with the moments of time that a clock cannot record. To each result of an operation of measurement with the clock corresponds a point in time: a moment $\theta(t)$, but there are, and this presupposition is essential in the present construction, points in time $\theta'(t)$ which do not correspond to an operation of measurement with a clock. The set of all these points in time $\theta(t)$ and $\theta'(t)$ is TIME. Without any further appeal at this point to mathematical demonstrations and their possible relevance, it is accepted that both the measurement of time with its limitations and time as such are continuous.

Dedekind cuts[2]

Dedekind cuts[3] are a way to build the real numbers **R** (**R** set) -the rational and irrational numbers- starting from the rational numbers set $\mathbf{Q}$. Further, it will be given a mathematical characterization of them that agrees to the present construction.

A Dedekind cut represents a *partition* of the rational numbers set in two sets M and N so that are satisfied the following requirements:[4]

1.) $M \subset \mathbf{Q}, N \subset \mathbf{Q}$

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[2] The next intuitive presentation follows, broadly, the text from ”en.wikipedia.org/wiki/**Dedekind_cut**” and Breaz Simion, Covaci Rodica - *Elemente de logică, teoria mulțimilor și aritmetică*, Editura Fundației pentru Studii Europene, Cluj-Napoca, 2006 and Purdea I., Pop I., *Algebră*, Editura Gil, Zalău, 2003

[3] Dedekind cuts are named after the German mathematician Richard Dedekind (he invented them).

[4] The definition of the partition of a set is given in a lower reference.

2.) $M \neq \varnothing, N \neq \varnothing$, M and N are non-empty sets.

3.) M contains no greatest element.

4.) $\forall x \in M \wedge \forall y \in N \Rightarrow x < y$

With this requirements, the Dedekind cut will be represented simply: (M, N).

There are two possibilities (cases):

1. If the set N has a smallest element "q" among the rational numbers $\mathbf{Q}$ then the cut corresponds to that rational number. In this case, the sets which define the Dedekind cut are: $M = (-\infty, q)$ and $N = [q, +\infty)$, and we say that the rational number q is represented by the cut (M, N)). In other words, if the set N has a smallest element among the rational numbers then the cut represents that rational number.

2. If the set N contains no smallest element among the rational numbers then the cut defines a unique irrational number which, intuitively speaking, fills the *gap* between the two sets of rational numbers: M and N. The sets which define the Dedekind cut in this case are: $M = (-\infty, q)$ and $N = (q, +\infty)$. In this way, M contains all rational numbers *to* the "break" between the sets of rational numbers M and N, next, N contains all rational numbers *starting from* the "break". ($M \cup N = \mathbf{Q}$). In other words, suggestively but still inadequate, M contains all rational numbers "smaller than" the gap (cut), and N contains all rational numbers "bigger than" the cut (gap).

The formulation is inadequate because it was not yet defined a partial order relation which encompasses both rational numbers and irrational numbers (in as will be the real numbers set and its order relation). Thus, a certain Dedekind cut can be associated with a "mathematical existence" which denotes an irrational number that doesn't belong to one of the two sets M and N covering the set of rational numbers $\mathbf{Q}$ $(\mathrm{M} \cup \mathrm{N} = \mathbf{Q})$. The real numbers set is equivalent with the set of all Dedekind cuts of $\mathbf{Q}$. In this construction each real number, rational or irrational number, corresponds to a unique cut.

"Whenever, then, we have to do with a cut produced by no rational number, we create a new, an irrational number, which we regard as completely defined by this cut; we shall say that the number corresponds to this cut, or that it produces this cut. From now on, therefore, to every definite cut there corresponds a definite rational or irrational number, and we regard two numbers as different or unequal always and only when they correspond to essentially different cuts."[5]

Complementary to this intuitive introduction for Dedekind cuts, we present an abstract definition of them in the Zermelo-Fraenkel axiomatic system.[6]

A *cut* is a subset $r \subseteq \mathbf{Q}$, together with the following properties:

(1) $r \neq \emptyset$;

(2) $r \neq \mathbf{Q}$;

(3) $q \in r \wedge p < q \Rightarrow p \in r$;

(4) r conteins no greatest element.

Analysing the results of this definition we remark that it is compatible with the above presentation. The result is a new number set: the

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[5] www.gutenberg.org/files/21016/2106-pdf.pdf: Dedekind Richard - *Essays on The Theory Of Numbers I. Continuity and Irrational Numbers II. The Nature and Meaning of Numbers*, authorised translation by Wooster Woodruff Beman Professor Of Mathematics in The University of Michigan Chicago, The Open Court Publishing Company London Agents Kegan Paul, Trench, Trübner & Co., Ltd. 1901, *Continuity and Irrational Numbers*, Section IV: *Creation of irrational numbers*, p. 7

[6] Breaz Simion, Covaci Rodica - *Elemente de logică, teoria mulțimilor și aritmetică*, Editura Fundației pentru Studii Europene, Cluj-Napoca, 2006, p. 148.

real numbers set. In the sense presented here, the Dedekind cuts among the real numbers may be considered as cuts among the rationals. It can show that every cut of real numbers set is identical to the cut produced by a certain real number which can be identified as the smallest element of the N set.

In the geometric representation, the ”number line” of real numbers, intuitive, is a picture of a straight line on wich every point corresponds to a real number and every real number corresponds to a point. In the context introduced above, every point (real number) on the number line is defined as a Dedekind cut of the rational numbers set and is a continuum without any gap. The intuitive picture is that when two straight lines cross, one is said to cut the other. The Dedekind’s construction of the number line ascertains us that the set of crossing points is not empty. The two straight lines have always one point in common: each of them does determine (does define) a Dedekind cut on the other.

The sets M and N define simmetrically a Dedekind cut because each of them does determine the other. With these significations it can be introduced a simplicity of the language. If we take arbitrarily as reference set the M set, the set without the greatest element, this set M can be defined as a Dedekind cut, but only in the next cases (which are self-understood): 1. $M = (-\infty, q)$ and $N = [q, +\infty)$ and 2. $M = (-\infty, q)$ and $N = (q, +\infty)$.

The intuitive concept of ”ordering” is represented by a partially ordered set. A partially ordered set is a set together with a binary relation. This indicates that, for certains pairs of elements in the set, one of the elements precedes the other element. The partial order reflects the fact that not every pair of elements need be ”order-related”. There are some pairs of elements that neither element precedes the other one in the set. The partial order generalize the total order in wich every pair of elements are ordered-related.

An important property of certain ordered sets Q is ”completeness”: every nonempty subset Q'(of the Q set) that is bounded above has a supremum that is also an element of Q.

The rational numbers set $(\mathbf{Q},\le)$ is a non-empty ordered set. More, $(\mathbf{Q},\le)$ is a totally ordered set. More, it is a lattice: for $\forall q_1, q_2 \in \mathbf{Q}, \exists \inf_{\mathbf{Q}}\{q_1, q_2\} \in \mathbf{Q} \wedge \exists \sup_{\mathbf{Q}}\{q_1, q_2\} \in \mathbf{Q}$. But the rational numbers set $(\mathbf{Q},\le)$, in the sens of ”completeness” defined above, is ”incomplete”. It is not a complete lattice because a complete lattice is a partially ordered set in which all subsets have a supremum in the partially ordered set.[7]

A classical example is this: $\sup\{q \in \mathbf{Q} | q^2 < 2\} \notin \mathbf{Q}, \left[\sup\{q \in \mathbf{Q} | q^2 < 2\} = \underset{\in \mathbf{R}}{\sqrt{2} \notin} \mathbf{Q}\right]$. Interpreting this example, the supremum of a subset of rational numbers is not a rational number [it is an irrational number ″(real)], which means that the rational number $(\mathbf{Q},\le)$ is ”incomplete”.

Let $(Q,\le)$ be an ordered set (particularly, the rational numbers set $(\mathbf{Q},\le)$, and $Q' \subseteq Q$. Let $\begin{cases} (i)\, q \in Q, \forall q' \in Q' \Rightarrow q' \le q \\ (ii)\, q_Q \in Q \wedge (\forall q' \in Q', q' \le q_Q) \Rightarrow q \le q_Q \end{cases}$ be two conditions. In this case ”q” is called supremum (or the least-upper-bound) of the Q' set in Q set: $q = \sup_Q Q'$. One may define the supremum for any subset of a partially ordered set. In this case, we say that respective set has

[7] In fact, the notion of complette lattice generalizes the supremum property of the real numbers.

the least-upper-bound [supremum] property if every subset of it with the property (i) also has the property (ii).[8] The supremum property is, for certain ordered sets, a fundamental property in mathematics. ”The supremum property” is another form of the completeness axiom (or the axiom of Cantor-Dedekind). It is intimately related to the construction of the real numbers using Dedekind cuts. The example above, indicates that the rational numbers set $(\mathbf{Q},\leq)$ does not have the supremum property under the usual order.

The construction of the real numbers using Dedekind cuts has the advantage of defining the irrational numbers as the supremum of certains subsets of the rational numbers. Moreover, the important feature of Dedekind cuts is to operate with number sets that are not complete.

The cut itself can represent a number which does not exist in the original set, in this case the rational numbers set $(\mathbf{Q},\leq)$. It is very important the fact that a cut (simbolized by *c*) can represent a new number, even though among the numbers belonging to the sets M and N we do not find the number *c*.

Relative to previous example, if the sets M and N contain rational numbers only, they can be cut at $\sqrt{2}$ putting every negative rational number together with every positive rational numbers whose square is, strictly less than 2, in the set $M = \{q \in \mathbf{Q} \mid q \geq 0 \Rightarrow q^2 < 2\}$ and, similarly, putting every rational number whose square is greater than or equal to 2, in the set $N = \{q \in \mathbf{Q} \mid q > 0 \wedge q^2 \geq 2\}$. $\sqrt{2}$ is not a rational number. In this way, all the rational numbers $(\mathbf{Q},\leq)$ are partitioned into two sets M and N (with

[8] The so called ”supremum property” is in fact equivalent to ”the completeness axiom”.

$M \cap N = \varnothing$), the partition itself signifying a new number (an irrational number).[9]

For our interests it is important the fact that we can order a Dedekind cut (M', N') as less than another Dedekind cut (M, N) by the instrumentality of subset and inclusion. The Dedekind cut (M', N') is less than the Dedekind cut (M, N) if M') is a subset of M ($M' \subseteq M$) or, equivalently, if N is a subset of N'($N \subseteq N'$). In this way, the inclusion relation of the sets ($\subseteq$) can be used to represent the ordering of Dedekind cuts, and the other relations ($\leq$) can be also constructed similary.

As well for our interests, it is important that the set of Dedekind cuts is a linearly ordered set. The construction by Dedekind cuts makes it possible the constraction of the real numbers.

An example of a continuous function, nowhere differentiable

Let be the next continuous function, nowhere differentiable: the Takagi-Knopp function.

Let H be a parameter, $0 < H < 1$ and let $g(t)$ be a periodic function with the period "1", $g(t)$ is defined on the interval $[0,1]$:

$$g(t) = \begin{cases} 2t, & 0 \leq t \leq \dfrac{1}{2} \\ 2(1 - t), & \dfrac{1}{2} \leq t \leq 1 \end{cases}$$

[9] Let S be a non-empty set.) It is called: *partition* of the set S a set P of non-empty subsets of S which satisfies the following conditions:

(i) $\varnothing \notin P$, P does not contain the empty set.

(ii) $S = \bigcup_i P_i$, The union of the elements of P is the set S.

(iii) $P_i \neq P_j \Rightarrow P_i \cap P_j = \varnothing$, The intersection of any two distinct elements of P is empty.

We observe that the period is ”1”.

$$g(0)=2\cdot 0=0,\ g(1)=2(1-1)=0, g(0)=g(0+1)=0 \text{ and}$$

$$g\left(\frac{1}{2}\right)=2\cdot\frac{1}{2}=2\left(1-\frac{1}{2}\right)=1, 2t\neq 0 \text{ for } \forall t\in\left(0,\frac{1}{2}\right), 2(1-t)\neq 0 \text{ for } \forall t\in\left(\frac{1}{2},1\right)$$

We olso observe that the function is a linear function on each interval:

Let $K(t)$ be the Takagi-Knopp function:

$$K(t)=\sum_{n=0}^{\infty}2^{-nH}g\left(2^n t\right)$$

The Takagi-Knopp function $K(t)$ for $H=\frac{1}{2}$ is:

$$K(t)=\sum_{n=0}^{\infty}2^{-\frac{n}{2}}g\left(2^n t\right).$$

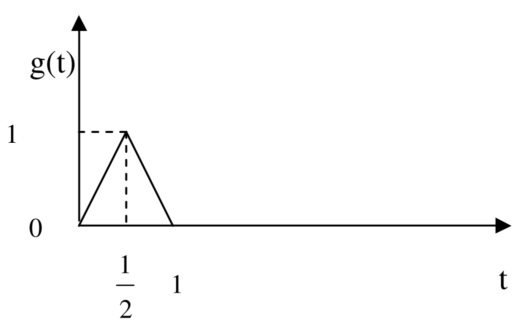
For a sum with a finite number of terms, the function $K_k(t)=\sum_{n=0}^{k}2^{-\frac{n}{2}}g\left(2^n t\right)$

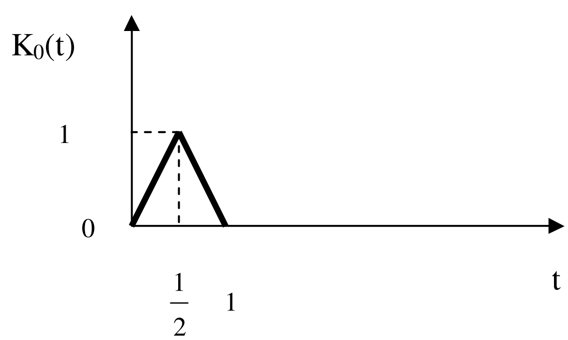
with $k+1$ terms is:

$$K_k(t)=\sum_{n=0}^{k}2^{-\frac{n}{2}}g\left(2^n t\right)=2^{-\frac{0}{2}}g\left(2^0 t\right)+2^{-\frac{1}{2}}g\left(2^1 t\right)+2^{-\frac{2}{2}}g\left(2^2 t\right)+\ldots+2^{-\frac{k}{2}}g\left(2^k t\right)=$$

$$=g(t)+\frac{\sqrt{2}}{2}g(2t)+\frac{1}{2}g(4t)+\ldots+\frac{\sqrt{2^{2-k}}}{2}g\left(2^k t\right)$$

For $k=0, K_0(t)=g(t)$. In this case, the graphical representation of the function $K_0(t)$ is the same with the graphical representation of the function $g(t)$:





For $k = 1, K_1(t)$ is a linear function on intervals:

$$K_1(t) = \sum_{n=0}^{1} 2^{-\frac{n}{2}} g(2^n t) = 2^{-\frac{0}{2}} g(2^0 t) + 2^{-\frac{1}{2}} g(2t) = g(t) + \frac{\sqrt{2}}{2} g(2t).$$

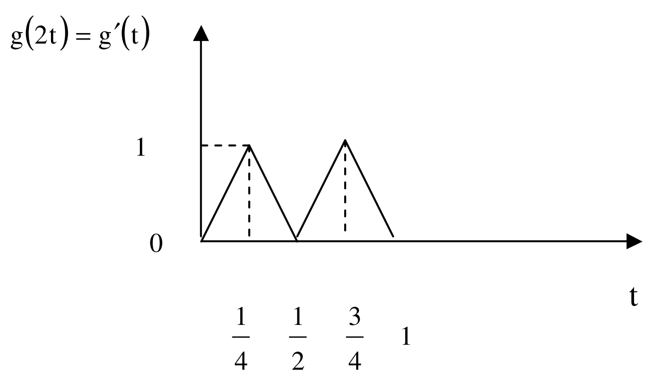
We observe that:

$$g(t) = \begin{cases} 2t, & 0 \le t \le \frac{1}{2} \\ 2(1 - t), & \frac{1}{2} \le t \le 1 \end{cases}$$

$$g(2t) = \begin{cases} 2(2t), & 0 \le 2t \le \frac{1}{2} \\ 2(1 - 2t), & \frac{1}{2} \le 2t \le 1 \end{cases} \Leftrightarrow g(2t) = \begin{cases} 4t, & 0 \le t \le \frac{1}{4} \\ 2(1 - 2t), & \frac{1}{4} \le t \le \frac{1}{2} \end{cases}$$

We observe that $g(2t) = g'(t)$ is a periodic function with the period ”$\frac{1}{2}$” and, consequently, for example: $g'\left(\frac{3}{4}\right) = g'\left(\frac{1}{4} + \frac{1}{2}\right) = g'\left(\frac{1}{4}\right) = 1$ și $g'(1) = g'\left(\frac{1}{2} + \frac{1}{2}\right) = g'\left(\frac{1}{2}\right) = 0$. We observe that $g(2t) = g'(t)$ is a periodic function with the period ” $\frac{1}{2}$ ”:

In this way, the graph of the function $g(2t)$ on the interval $[0,1]$ is:



For the functions of $K_1(t)$: $K_1(t)=g(t)+\frac{\sqrt{2}}{2}g(2t)$ it can be writed:

$$g(t)=\begin{cases} 2t\,, & 0\le t\le\frac{1}{2} \\ 2(1-t), & \frac{1}{2}\le t\le 1 \end{cases}$$

$$\frac{\sqrt{2}}{2}g(2t)=\begin{cases} 2\sqrt{2}\,t\,, & 0\le t\le\frac{1}{4} \\ \sqrt{2}(1-2t), & \frac{1}{4}\le t\le\frac{1}{2} \end{cases}$$

The complete representation of the function $K_1(t)$ on the interval $[0,1]$ is:

$$K_1(t)=g(t)+\frac{\sqrt{2}}{2}g'(t)$$

We have the next particular values for drawing the graph of the function $K_1(t)$:

$$K_1(0)=g(0)+\frac{\sqrt{2}}{2}g'(0)=0+0=0\,,$$

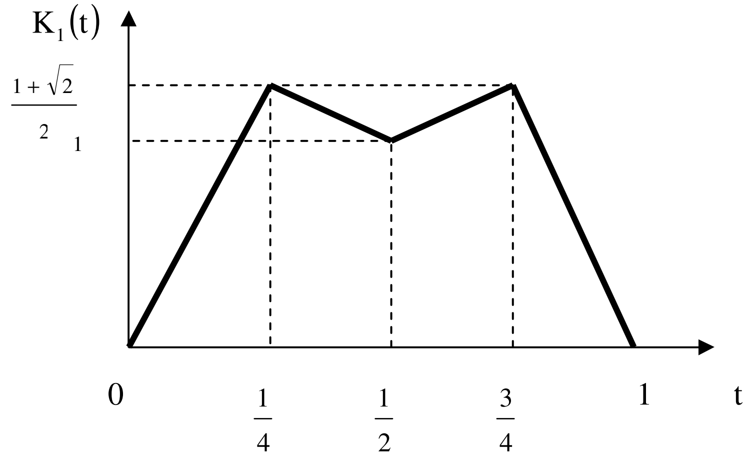
$$K_1\left(\frac{1}{4}\right)=g\left(\frac{1}{4}\right)+\frac{\sqrt{2}}{2}g'\left(\frac{1}{4}\right)=\frac{1}{2}+\frac{\sqrt{2}}{2}\cdot 1=\frac{1+\sqrt{2}}{2}\,,$$

$$K_1\left(\frac{1}{2}\right)=g\left(\frac{1}{2}\right)+\frac{\sqrt{2}}{2}g'\left(\frac{1}{2}\right)=1+\frac{\sqrt{2}}{2}\cdot 0=1\,,$$

$$K_1\left(\frac{3}{4}\right)=g\left(\frac{3}{4}\right)+\frac{\sqrt{2}}{2}\cdot 1=\frac{1}{2}+\frac{\sqrt{2}}{2}=\frac{1+\sqrt{2}}{2}\,,$$

$$K_1(1)=g(1)+\frac{\sqrt{2}}{2}g'(1)=0+0=0$$

The graph of the function $K_1(t)$ is:



For $k = 2, K_2(t)$ is also a linear function on intervals:

$$K_2(t) = \sum_{n=0}^{2} 2^{-\frac{n}{2}} g(2^n t) = 2^{-\frac{0}{2}} g(2^0 t) + 2^{-\frac{1}{2}} g(2^1 t) + 2^{-\frac{2}{2}} g(2^2 t) = g(t) + \frac{\sqrt{2}}{2} g(2t) + \frac{1}{2} g(4t)$$

It follows:

$$g(4t) = \begin{cases} 2(4t)\ , & 0 \le 4t \le \frac{1}{2} \\ 2(1-4t), & \frac{1}{2} \le 4t \le 1 \end{cases} \Leftrightarrow g(4t) = \begin{cases} 8t, & 0 \le t \le \frac{1}{8} \\ 2(1-4t), & \frac{1}{8} \le t \le \frac{1}{4} \end{cases} \text{ şi}$$

$$\frac{1}{2} g(4t) = \begin{cases} 4t, & 0 \le t \le \frac{1}{8} \\ 1-4t, & \frac{1}{8} \le t \le \frac{1}{4} \end{cases}$$

We observe that $g(4t) = g''(t)$ is a periodic function with the period ” $\frac{1}{4}$ ”

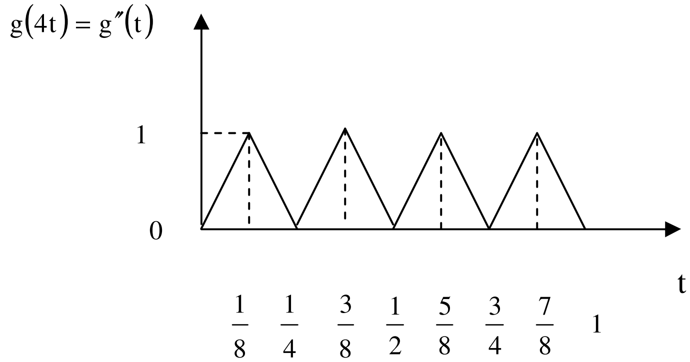
and, consequently, for example:

$$g''\left(\frac{3}{8}\right) = g''\left(\frac{1}{8} + \frac{1}{4}\right) = g''\left(\frac{1}{8}\right) = 1$$

and

$$g''(1)=g''\left(\frac{3}{4}+\frac{1}{4}\right)=g''\left(\frac{3}{4}\right)=g''\left(\frac{1}{2}+\frac{1}{4}\right)=g''\left(\frac{1}{2}\right)=g''\left(\frac{1}{4}+\frac{1}{4}\right)=g''\left(\frac{1}{4}\right)=g''\left(0+\frac{1}{4}\right)=g''(0)=0$$

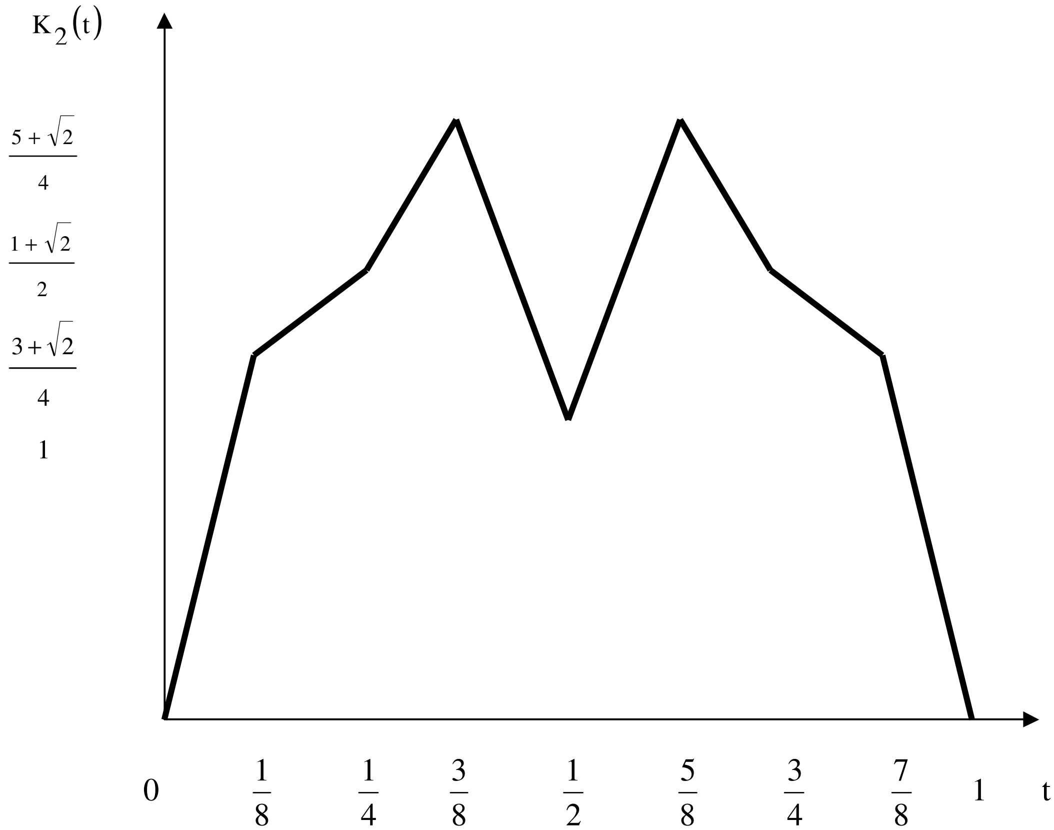
In this way, the graph of the function $g(4t)$ on the interval $[0,1]$ is:



We have in a table the next particular values for drawing the graph of the function $K_2(t)$ on the interval $[0,1]$:

t	0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1
$g(t)$	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	0
$\frac{\sqrt{2}}{2}g'(t)$	0	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{4}$	0	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{4}$	0
$\frac{1}{2}g''(t)$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0
$K_2(t)$	0	$\frac{3+\sqrt{2}}{4}$	$\frac{1+\sqrt{2}}{2}$	$\frac{5+\sqrt{2}}{4}$	1	$\frac{5+\sqrt{2}}{4}$	$\frac{1+\sqrt{2}}{2}$	$\frac{3+\sqrt{2}}{4}$	0

The graph of the function $K_2(t)$ is:

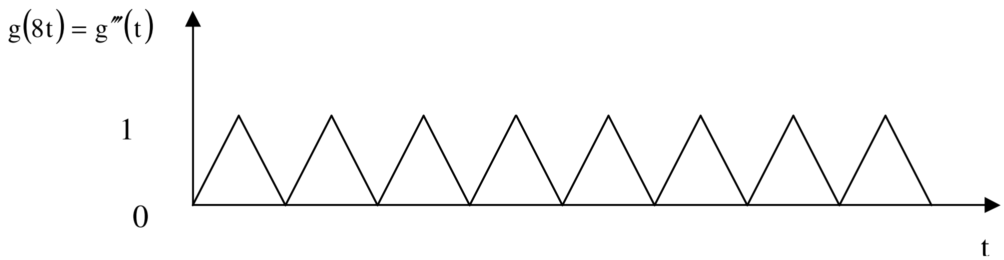


For $k = 3$ we shall only sketch the function $K_3(t)$:

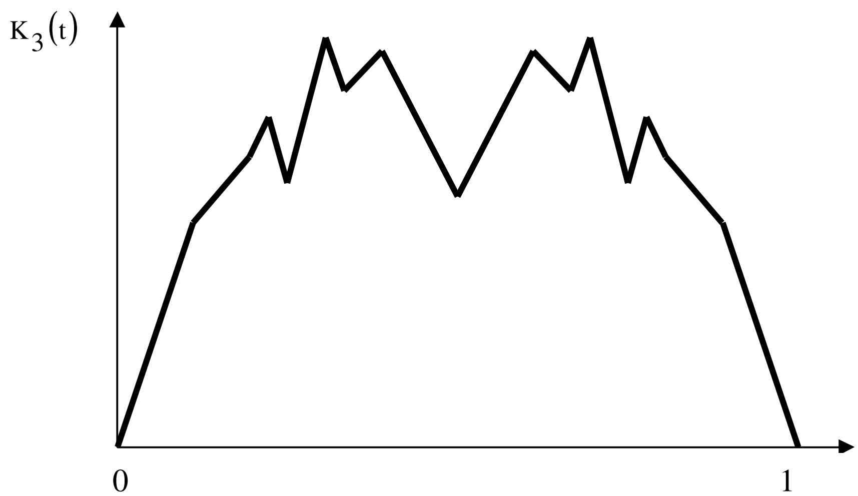
$$K_3(t) = \sum_{n=0}^{3} 2^{-\frac{n}{2}} g(2^n t) = 2^{-\frac{0}{2}} g(2^0 t) + 2^{-\frac{1}{2}} g(2^1 t) + 2^{-\frac{2}{2}} g(2^2 t) + 2^{-\frac{3}{2}} g(2^3 t) =$$

$$= g(t) + \frac{\sqrt{2}}{2} g(2t) + \frac{1}{2} g(4t) + \frac{\sqrt{2}}{4} g(8t)$$

The graph of the function $g(8t)$ on the interval $[0,1]$ is:



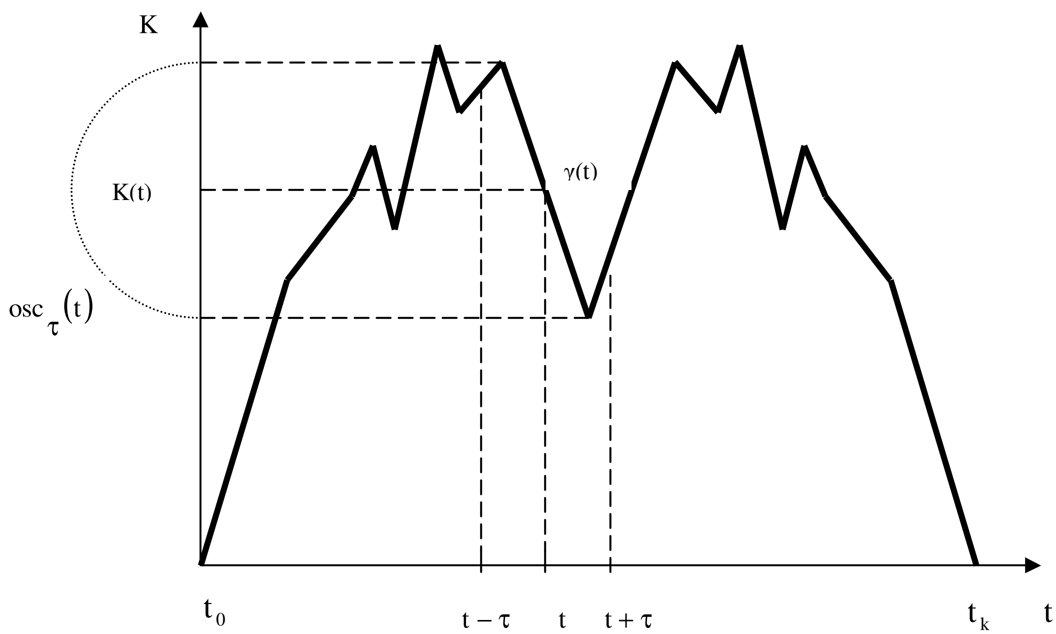
The graph of the function (the morphology) $K_3(t)$ on the interval $[0,1]$ is:



For $n \to \infty$ the fragmentation of the curve and, implicitely, its global irregularity of this, grow considerably. In the first graph (morphology), for n=0 there is no tangent line to the curve in one point only; for n=1 there are not tangents to the curve in three points; for n=2 there are not tangents to the curve in seven points; for n=3 there are not tangents to the curve in fifteen points etc., for $n \to \infty$ the Takagi-Knopp function remains a continuous function but nowhere differentiable.

The fractal dimension measuring the global irregularity of the curve is: $\Delta(\Gamma) = 2 - H$. *In the example above we considered* $H = \frac{1}{2}$ *and, consequently, the fractal dimension of the curve was* $\Delta(\Gamma) = 1.5$. *Let us mention that the irregularity of the curve changes with the change of the fractale dimension for* $H = 1, \frac{3}{4}, \frac{1}{2}, \frac{1}{4}$.

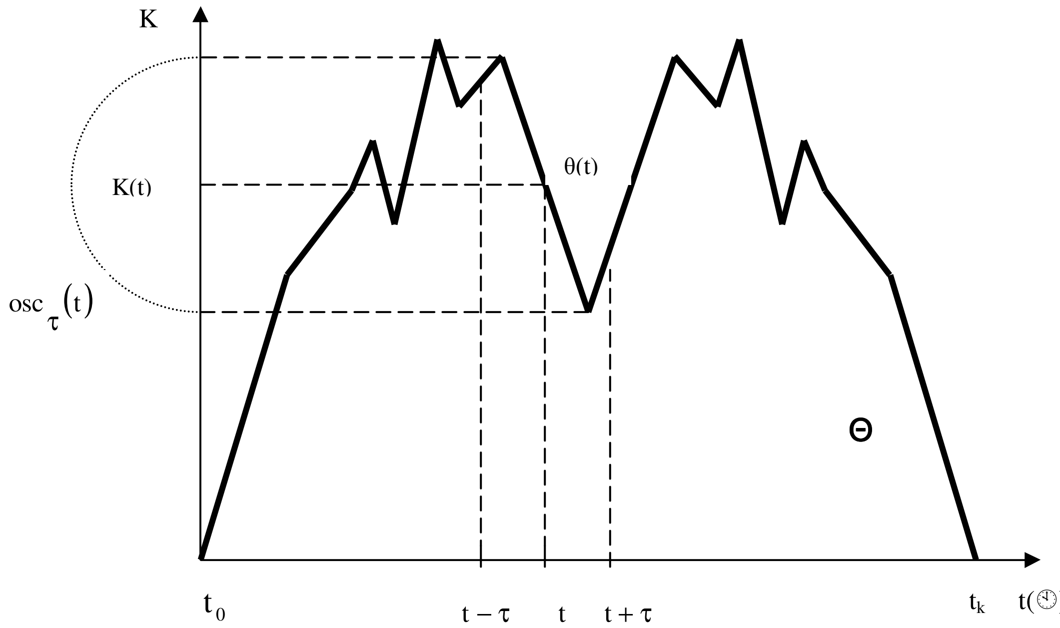
For a more simple mathematical exposure, the continuous function $K_3(t)$ will be generally considered to be representative, and it will be denoted: $K(t)$.



We shall consider a set of ordered pairs of points $(t, K(t))$ (t is for t(🕘)), in a plain. In this case, the abscissa t has the physical dimension of the time: we suppose that each Dedekind cut has this physical dimension even if the clock cannot measure it. The ordinate $K(t_i)$ is the value of a physical quantity[10] (a system state) in t_i point. We shall consider the morphology of the dynamics $K(t)$ as being the graph Γ of this continuous function on the interval $[t_0, t_k]$. We construct a function, the parametrization $\gamma(t):\ t \mapsto (t, K(t))$. We call *local arc* of parametrization (or "fragment of morphology") -noted $\gamma(t-\tau)\frown\gamma(t+\tau)$- the part of the graph Γ corresponding to the abscissas from the interval $[t-\tau, t+\tau]$; interval which represents the projection of the fragment of morphology $\gamma(t-\tau)\frown\gamma(t+\tau)$ on the Ot axis. The projection of the fragment of morphology on the OK axis is the interval $\left[\inf\ z(t)_{t\in[t-\tau,t+\tau]}, \sup z(t)_{t\in[t-\tau,t+\tau]}\right] = \mathrm{osc}_\tau(t)$.

The basic assumption of this analysis is the following: the time is exterior to the operation of physical measurement (performed by a clock and symbolized here by t(🕘)). In this way the above representation it will be understood like this:

[10] O mărime fizică este o proprietate fizică a unui fenomen care poate fi cuantificată prin măsurători (A physical quantity is a physical property of a phenomenon that can be quantified by measurements.)



The further requirement deriving from this formulation of the problem is the construction of a continuous function nowhere differentiable which should express as good as possible the "physical picture" of the current time.