# The Use of Excluded Middle in the Theological Discourse

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**Abstract:** This paper aims to expose the eventual advantages and the disadvantages of the excluded middle within the theological type of discourse according to the model of the intuitionist logic. The relinquishment of the law of the excluded middle (LEM) leads to the invalidity of the ontological argument in its various formalizations in the case of Saint Anselm because it is necessary the reduction to the absurd which becomes impossible once the LEM is rejected and in the Hartshorne, Godel and Platinga cases because S5 is requested, whilst the intuitionist logic can only be transposed in S4. By comparison, I will present some of the advantages and of the disadvantages of refuting the principle of the non-contradiction as it is depicted by the paraconsistent logic. Through this, I will also explain why theologians would be advantaged by a logic system which rejects the non-contradiction and that accepts the LEM.

**Keywords:** Ontological proof, excluded middle, non contradiction, paraconsistent logic, intuitionism, theological discourse

The paper starts with the observation that all the formulations of the ontological argument need to accept the principle of the excluded middle and therefore of the system S5 in the modal logic in order to ensure the validity of the argument. Moreover, I will present a few formalizations of the argument in order to prove this statement. The paper is not aiming to jump into another type of debates concerning the validity of the argument;

such debates are disseminated into the specific literature and overpass the limits of the present paper.

The classical ontological argument in Saint Anselm's version supposes a reduction ad absurdum judgment. Starting with the non-existence of God in reality and showing that it leads to an absurdity, the argument deduces the falsity of the premise from which it starts and, by consequence, the existence of God. od exists in the understanding, but not in reality. (Assumption for reduction)

1. Existence in reality is greater than existence in the understanding alone. (Premise)
2. A being having all of God's properties plus existence in reality can be conceived. (Premise)
3. A being having all of God properties plus existence in reality is greater than God. (From (1) and (2).)
4. A being greater than God can be conceived. (From (3) and (4).)
5. It is false that a being greater than God can be conceived.
   (Since the proper signification of "God")
6. Hence, it is false that God exists in the understanding but not in reality.
   (From (1), (5), (6).)
7. God exists in the understanding. (Premise)
8. Hence God exists in reality. (From (7), (8).

So, if God exists in the nous, than He exists also in reality, but it is as egregious as possible that He exists in the intellect and so He exists in reality too[1].

Let's consider Hartshorne's formalization of the ontological argument[2].

Premise

1. If there is a perfect being then a perfect being is necessary to exist.
2. It is possible that a perfect being may exist.

[1] Platinga, A., 1998, *Natura Necesității*, Trei, p. 303-304.
[2] Miroiu, A., 2011, *Argumentul ontologic*, ALL, p.316-317.

We note with q the expression: "there is a perfect being", formalized as ($\exists$ **x**) P(x)

Demonstration

1. q→ □q                                   Premise
2. ◊q                                       Premise
3. ◊q → ◊ □q ( from (1) based on the principle (a→ b)→ (◊ a → ◊b) )
4. ◊ □q → □q (defining principle in S5)
5. ◊ □q          (from ( 2) and (3))
6. □q            (from (4) and (5))
7. □q →q      (principle of the modal logic)
8. q             (from(6) and (7) )

One wanting to contest the ontological argument has two alternatives: 1- either contesting the premises or 2- contesting the valid passing from one to another. Or, in order to demonstrate the validity of the ontological argument there is a need for S5[3]. The intuitionist logic is not equivalent to S5. All the intuitionist axioms and theorems are theorems in S4[4]. The previous argument cannot be deduced from S4 because from the possibility of needing an object we can deduce the necessity of the respective object only in S5 (◊ □q → □q).

The same demonstration of the argument in S5 is present in Platinga's formalization.

A being has maximal excellence in a given possible world W if and only if it is omnipotent, omniscient and wholly good in W; and

1. A being has maximal greatness if it has maximal excellence in every possible world.
2. It is possible for a being to have maximal greatness. (Premise)
3. Therefore, it is possible necessarily true that an omniscient, omnipotent and perfectly good being may exist.
4. Therefore (by axiom S5) it is necessarily true that an omniscient, omnipotent and perfectly good being exists.

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[3] Miroiu, A, 2011, *Argumentul ontologic*, ALL, p 316-317
[4] Dumitriu, A, 1971, *Logica polivalentă,* Bucureşti, p. 244

Therefore, an omniscient, omnipotent and perfectly good being exists[5].

Formalizing the ontological argument in the vision of Gödel

It is possible to also consider another formalization of the ontological argument, the one advanced by Gödel. In this case, the premise has the following content: for any property A, either A is positive or non A is positive. Thus we have a new use of the principle of the excluded middle, even more obvious than the one having been expressed earlier.

Definition 1: x is God-like if and only if x has as essential properties those and only those properties which are positive.

Definition 2: A is an essence of x if and only if for every property B, x has B necessarily if and only if A entails B.

Definition 3: x necessarily exists if and only if every essence of x is necessarily exemplified.

Axiom 1: Any property entailed by—i.e., strictly implied by—a positive property is positive.

Axiom 2: If a property is positive, then its negation is not positive.

Axiom 3: The property of being God-like is positive.

Axiom 4: If a property is positive, then it is necessarily positive.

Axiom 5: Necessary existence is positive.

Axiom 6: For any property P, if P is positive, then being necessarily P is positive.

Theorem 1: If a property is positive, then it is consistent, i.e., possibly exemplified.

Corollary 1: The property of being God-like is consistent.

Theorem 2: If something is God-like, then the property of being God-like is an essence of that thing.

Theorem 3: Necessarily, the property of being God-like is illustrated through a thing.[6]

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[5] Plantinga, A, 1998, *God, arguments for the existence of*, in E. Craig (Ed.), *Routledge Encyclopedia of Philosophy,*London: Routledge.

[6] http://plato.stanford.edu/entries/ontological-arguments/#GodOntArg, accessed on 03.11.2011

All the four builds of the ontological argument prove that we need the principle of the excluded middle in its formulation, as in the case of any other judgment which would suppose reduction ad absurdum. The aim of this demonstration is to prove that once it is impossible to hold an ontological argument, it is also impossible to hold a reasonable discourse about God. This does not mean that He does not exist, but just that the attempt of defying Him by recalling the human reason is an invalid process. By consequence, it is not surprising at all that the founder of the intuitionism proposes, along texts from his youth, a mystical approach over the transcendental truth; this does go in parallel with the rejection of the present infinite. "Mysticism simply denies that there is nothing positive to be found in this limited life[7]". The transcendental truth is considered existing but hardly accessible and more often in a partial and only in a suggested manner.

> "Sometimes only the accompaniment of transcendent truth may be heard in life, truth itself is absent, remains outside this limited life and therefore outside the domain of communication and mutual understanding[8]".
> "In expression, transcendent truth cannot be revealed-even less than immanent truth without causing an outrage. A clear statement of truth, seriously and emphatically pronounced is no more acceptable that the manifest performance of miracles[9]".

Therefore we are facing during this lifetime only a copy of the truth, a "knowledge through guessing" of it as Saint Paul states.

> „Writings of transcendent truth which have been preserved are usually the work of an imitator, their real, spiritual father never had the inclination to write, he radiated truth throughout his life, infinitely stronger than he could ever express in words or in writing. He was never able to water down the

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[7] Brouwer, "Life, Art and Mysticism", 1905, *Notre Dame Journal of Formal Logic*, Volume 37, Number 3, Summer 1996, p. 418, visited on http://projecteuclid.org/DPubS/Repository/1.0/Disseminate?view=body&id=pdf_1&handle =euclid.ndjfl/1039886518 , accessed on 03.11.2011
[8] Brouwer, "Life, Art and Mysticism",  p 418
[9] Brouwer, J 1905,"Life, Art and Mysticism", p 414

truth as society demands, neither did he feel inclined to allow the truth, rising so high above the world to go begging here on earth, allow truth, which transcends language, to go begging in words. [10]

It is natural that such an approach- that places the transcendental truth beyond the idiom and the reason- to consider irrelevant and irony worthy any attempt to speak about God in intellectual terms. "It is not more than a little game when so-called philosophers start giving rational explanations of God, Trinity, Immaculate Conception etc."[11]

I have extracted these few fragments from Brouwer because they are a good illustration of the effects of rejecting the excluded middle in the theological discourse belonging to the founder of the intuitionism.

The discourse which overlooks the excluded middle has as consequence the exclusion of the proper use of the reason in the theological discourse. God remains impossible to be known at a reasoning level. In an analogous manner, for the intuitionist logic the infinite is not actual, it is just potential, the same as for a novel before being written, the brouwerian successions are not unlimited but they strive to infinite. That is to say they have only a way of continuously envisaging a new element. The principle of the excluded middle cannot apply to infinite sets. This intuitionist strategy avoids the paradoxes where the formalists fell. But it is exactly the paradoxes that are the approach through which the theology has attempted to speak about God. If we are to no longer accept the principle of the excluded middle, then its formulations about God will still not appear paradoxical, they will simply turn into some assertions which operate with the infinite; therefore they will no longer be included in the sum of the sets that operate with the principle of the excluded middle.

The classical theologians, both those from the West and from the East had the claim of speaking about God to the extent of the human understanding, therefore the paradoxes appear natural when one talks about the relevant essence of God. They do not reject the excluded middle principle; in exchange they reject that of the non-contradiction in the sole

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[10] Brower, "Life, Art and Mysticism", p 415.
[11] Brouwer, "Life, Art and Mysticism", p 419.

case of talking about God. Let us consider an example of rejection of the non-contradiction principle within the Divinity discourse. I have chosen the case of Cusanus because he identifies the notion of God with the notion of mathematical infinite. Remains to check in which way Cusanus operates with the notion of infinite and in which manner he proves the way in which the finite logical cannot be applicable in the case of the Creator.

He moreover provides some shocking identities that seem paradoxical but if we accept the definitions provided by Cusanus, they are not so. Cusanus demonstrations are anti-intuitive. The unique thing to reproach to Cusanus is that his demonstrations are determined by the existence of an actual infinite and by the belief that there is only one infinite. But, if we are to accept the fact that there is only one infinite that exists, then it results that the infinite line is in fact an infinite triangle. Totally otherwise happens with things belonging to the non-maximal field, because in that case the potency is not an act and the infinite line is not a triangle,[12] as he explains to us. Briefly resuming, his conception about the infinite states as follows: if there was an infinite line, it would turn successively into a straight line, a triangle, a circle, and an orb. Similarly, if there is an infinite orb, it will turn into a circle, a triangle and a line[13]. Below we will take an example from the reasoning method of Cusanus: The infinite line is a triangle.

> „Since in the case of quantitative things a line and a triangle differ incomparably, the imagination, which does not transcend the genus of perceptible things, does not apprehend that the former can be the later. However this will be easy for the intellect. It is already evident that there can be one maximum and infinite thing. Moreover, since any two sides of any triangle cannot, if conjoined, be shorter that the third: it is obvious that in the case of a triangle whose one side is infinite, the other two sides are not shorter (i.e. they are together infinite). And because each part of what is infinite is infinite: if any random side of a triangle is infinite, the other sides must also be infinite. And since there cannot be more that one infinite

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[12] Cusanus, N, 2008, *De docta ignorantia,* Polirom, p. 109 ( for my one use I read the romanian version, for quatation i used the traslation of Hopkins, J)

[13] Cusanus, N, 2008, *De docta ignorantia*, Polirom p. 93

thing, you understand transcendentally that an infinite triangle cannot be composed of a plurality of lines, even though it is the greatest and truest triangle, incomposite and most simple. And because it is the truest triangle-something which cannot be drawn without three lines- it will be necessary that the infinite line has three lines and that the three lines be a most simple line"[14].

The cardinal demonstrates, in the same manner precisely, in which way the triangle with an infinite side represents a circle and in which way the circle turns into an orb. The purpose of these examples is to talk about God through the mathematics language and to show that both in the case of Divinity -who represents the 'Absolute maximum which is identical to the absolute minimum'[15] and in the case of the geometrical infinity one cannot judge the non-contradictory terms that the human mind is familiarized to operate with.

The contradictory type of expression is frequently present in the dogmatic theology when speaking about God. The Calcedon Council settles that Christ is God and Man at the same time, that He has two kinds, united without confusion.

„ One is Christ out of two kinds, of Divinity and of humanity... because being a perfect God and a perfect human he naturally worked through both of those for which He was state without dividing in between them[16]".
„We shall declare Christ true God and true man, that One being in reality, in the proper sense... and being truly both... and so having two kinds and two wills[17]".

The two quotations from Saint Maxim the Confessor are chosen in order to illustrate the habitual way of expressing oneself in theology, in order to prove that the theological discourse uses the special case of

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[14] Cusanus,N *De docta ignorantia*, I-chp 14, p 22 transtated by Hopkins, J, Librabry of the Congres, 1985, visited on http://jasper-hopkins.info/DI-I-12-2000.pdf in data de 28.11. 2011 accessed on 12.11.2011
[15] Cusanus,N, De docta ignorantia, I-chp 4, p. 9
[16] Maxim the Confessor, 1990, *Scieri, Partea a II a,* Editura Insititutului Biblic, p. 192-193
[17] Maxim the Confessor,1990, *Scieri, Partea a II a*, Editura Insititutului Biblic p. 195

eliminating the non-contradiction principle in the sole case of the discourse upon Divinity.

The stake of the above examples is to prove that in the case of the infinite it is impossible to apply the same reasoning way as in the case of the finite. When speaking about infinite at least one of the principles of the classical logic fails. The question that rises would be to which principle would be more advantageous to give up in the theological discourse? Would it be to the excluded middle or to the non-contradiction principle? Given the already stated examples, it seems that the refutation of the excluded middle principle would throw one into an agnostic situation when one cannot set forth anything reasonable about infinite. On contrary, the refutation of the principle of non-contradiction, as it happens both in the case of Cusanus and of Saint Maxim, manages to deliver an informational content, to still suggest an image about God. To say that God is both the maximum and the minimum, that Christ is both God and man provides more information about Divinity than saying that He cannot be comprised in expression, that we cannot talk about Him, that we cannot express through language the transcendental truth as Brouwer affirms.

Let us move forward in the attempt of finding a formal system to preserve the excluded middle principle and to repel the principle of non-contradiction. A remarkable thing to observe with respect to the rejection of the excluded middle principle is the fact that the majority of the systems of paraconsistent logic accept the excluded middle principle and a non-universal form of the reduction ad absurdum principle. We shall recall that the paraconsistent logic principles repel the classical idea according to which from false could result anything (i.e., *ex falso quodlibet*). Therefore, for the paraconsistent logic

$$\neg \,((\,A\,\&\,\neg\,A)\,\rightarrow B\,).$$

One of the simplest paraconsistent logic systems is that of Newton de Costa which accepts that p V ¬ p but repels that p & ¬ p. Furthermore, it accepts that ¬¬ A→A but it repels that A→¬¬ A, exactly on the opposite way then the intuitionist logic. It is still interesting to observe how through the acceptation of the principle of the excluded middle can be formulated

the conditions for drawing up an argument through reduction *ad absurdum*[18].

> We define B&¬B as being B˚.
> If from Γ and A is deduced B˚    and
> If from Γ and A is deduced B     and
> Then if from Γ and A is deduced ¬B
> Then from Γ is deduced ¬ A

The discussion that Graham Priest[19] holds around accepting the reasoning through reduction *ad absurdum* in the paraconsistent logics is worthy of consideration. For him, an argument by reduction can be an efficient one, only that it is not a necessary and a universal one. Sometimes we re-feel the constraint of rejecting contradictions, the supporters of the paraconsistency do not involve into accepting all the contradictions. An advocate of the paraconsistency may seriously accept both alternatives, both the rejection and the acceptation of a contradiction; still, in cases taken hazardously, it is very possible to reject contradictions. Under the condition of accepting A V ¬ A and of repelling the paradoxical nature of the premises, we can admit the conclusion of a formal demonstration which employs the reduction *ad absurdum*. This conclusion will surely sound in the following way: if Σ represents any deductive closed system of enunciations, Σ is locally consistent with respect to B, if and only if B ∈ Σ or if ¬B ∈ Σ.   Σ is locally inconsistent to B if and only if Σ is not consistent with respect to B. Therefore, the reduction *ad absurdum* may be acceptable, even though quasi- valid, under the condition of accepting the principle of the excluded middle and by reasonably repelling a local inconsistency.

I have attempted to prove that the logical system accepting the excluded middle principle and rejecting the non-contradiction principle in isolated cases represents an advantage for the theological type of discourse,

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[18] Lucica, I, *Logica şi filosofia contradicţiei. Incursiuni în subiectul paraconsistenţei*, in Lucica,I,  Gheorghiu, D, Ghirilă, R *Ex Falso quod libet, Studii de logică paraconsistentă,* Editura tehnică, 2004, p. 13

[19] Priest,G, „Reductio ad absurdum şi modus tollendo ponens", in Lucica, I, Gheorghiu, D, Ghirilă, R, *Ex Falso quod libet, Studii de logică paraconsistentă,"*2004, Editura tehnică, p. 393 .

available in the cases when discussing on infinite preferably. The version inspired by the intuitionism repelling the excluded middle presents two major disadvantages: a- the invalidity of the ontological argument and b- the impossibility of talking with an end about the infinite in acting. Finally, I would like to highlight that the acceptation of the ontological argument does not turn one into a Christian as the Christianity supposes a reference to a personal God. Lots of theologians refuse to talk about God in the terms of the classical philosophy precisely because escaping the exposure to identify in a fallacious manner the God of the philosophers with the God of Abraham and of Jacob.