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Babeş-Bolyai University of Cluj-Napoca
Faculty of History and Philosophy
Department of Philosophy

SELFLARINC's Final Conference

Incompleteness, New Axioms and Truth

5-6 December 2025

PROGRAMME AND BOOK OF ABSTRACTS

Organisers:

Adrian Ludusan
Mihai Rusu
Claudio Ternullo

PROGRAMME

5 December

Morning

10:00-10:15 Organisers: *Greetings*
10:15-11:15 Joan Bagaria: *Large Cardinals Beyond HOD*

11:15-11:45 *Coffee Break*

11:45-12:45 David Asperó: *Around Reinhardt Cardinals*

12:45-14:15 *Lunch Break*

Afternoon

14:15-15:15 Beau Madison Mount: *The New Large Cardinals*
15:15-16:15 Cesare Straffelini: *The Inevitability of Determinacy*

16:15-16:45 *Coffee Break*

16:45-17:45 Monroe Eskew: *The Humility Principle*

20:00 **Social Dinner 1**

6 December

Morning

10.00-11:00 Antonio Piccolomini d'Aragona: *Types, sets, and models in a Kuhnian-Lakatosian perspective*
11:00-12:00 Giorgio Venturi: *Per aspera ad astra: from Skolem's Paradox to an uncountable universe*

12:00-12:30 *Coffee Break*

12:30-13:30 Xinhe Wu: *Boolean-Valued Models and Urelements*

13:30-14:30 *Lunch Break*

Afternoon

14.30-15.30 Adrian Ludusan: *On the completeness interpretation of representation theorems*
15:30-16:30 Deborah Kant: *The Foundational Role of Set Theory and Set-Theoretic Pluralism*

16:30-17:00 *Coffee Break*

17:00-18:00 Michał Godziszewski: *Tennenbaum's Theorem for quotient presentations and model-theoretic skepticism*

20.00 **Social Dinner 2**

Venue (both days): Aula Regele Ferdinand I (National History Institute), Str. Napoca, 11

ABSTRACTS

David Asperó (University of East Anglia)

Around Reinhardt Cardinals

I am planning to present some results involving Reinhardt cardinals. One result concerns the construction of inner models of ZFC with large cardinals in the presence of a Reinhardt cardinal, and another result involves a forcing construction for blowing up Lindenbaum numbers while lifting Reinhardt embeddings. The second result is joint work with Eric Nichols. I will also argue for the foundational import of these results.

Joan Bagaria (University of Barcelona, ICREA)

Large Cardinals Beyond HOD

There is a new, recently discovered, dividing line in the consistency hierarchy of large cardinals. Namely between those compatible with the strong form of the Axiom of Choice given by the axiom V=HOD, which asserts that every set is hereditarily definable with ordinal parameters, and those incompatible with it, yet still compatible with the Axiom of Choice. The new large cardinals beyond HOD pose a new challenge to our understanding of large cardinals. For one thing, all inner models for large cardinals studied so far by the inner model program satisfy V=HOD, and so the new large cardinals fall beyond the range of current inner model theory. Moreover, they interact in unexpected ways with current large cardinals, calling into question the linearity of the consistency hierarchy. Furthermore, they suggest that large cardinals should be studied not only on the basis of their consistency strength, but also based on the degree to which they imply failures of different forms of the Axiom of Choice. A foundational question remains: to what extent the new large cardinals beyond HOD are genuine large cardinals?

Monroe Eskew (University of Vienna)

The humility principle

We will argue that a liberal reading of Gödel's philosophical writings suggests a third kind of justification of axioms beyond the usual intrinstic/extrinstic distinction. This third kind has to do with identifying general truths about mathematics and logic and its relation to ourselves, which can be partially empirical. We will argue for one such truth, the Humility Principle (HP), which characterizes a contrast between mathematical richness and the ability of logical systems to capture it. We will then argue that large cardinals and some of their generic cousins are justified by the HP. The generic large cardinals we focus on settle many set-theoretic questions.

Deborah Kant (Free University of Brussels)

The Foundational Role of Set Theory and Set-Theoretic Pluralism

The foundational role of set theory is often tied to a universe view: since set theory provides the foundation of mathematics, it must in principle be able to answer all mathematical questions, including those requiring new axioms beyond ZFC. Against this background, set-theoretic pluralism seems to threaten the foundational role—if there are multiple legitimate extensions of ZFC, some mathematical questions may lack determinate answers.

In this talk, I challenge the assumption that the foundational role of set theory is necessarily linked to the universe view. My argument draws on two key observations about mathematical practice. First, set theory is a relatively isolated field: contemporary research rarely interacts with other areas of mathematics, with exceptions such as Farah's result on the Calkin algebra. Second, many pluralist set theorists nonetheless regard set theory as foundational. Building on the first observation, their position rests on a compelling point: mathematics outside set theory provides sufficient justification for the ZFC axioms, but not for any axioms beyond ZFC.

This yields a coherent justification of set-theoretic pluralism: if set theory must provide a foundation for mathematics, and ZFC fulfills this role, then set-theoretic pluralism is well grounded. The universist, in turn, faces two options: either justify the universe view on purely set-theoretic grounds, rather than through its foundational role, or demonstrate that new axioms are needed in mathematics to the same extent as, for example, the Axiom of Choice.

Michał Godziszewski (University of Warsaw)

Tennenbaum's Theorem for quotient presentations and model-theoretic skepticism

A computable quotient presentation of a mathematical structure A consists of a computable structure on the natural numbers $\langle N, \star, *, \cdots \rangle$, meaning that the operations and relations of the structure are computable, and an equivalence relation E on N, not necessarily computable but which is a congruence with respect to this structure, such that the quotient $\langle N, \star, *, \cdots \rangle$ is isomorphic to the given structure A.

Thus, one may consider computable quotient presentations of graphs, groups, orders, rings and so on. A natural question asked by B. Khoussainov in 2016, is if the Tennenbaum Thoerem extends to the context of computable presentations of nonstandard models of arithmetic. In a joint work with J.D. Hamkins we have proved that no nonstandard model of arithmetic admits a computable quotient presentation by a computably enumerable equivalence relation on the natural numbers. However, as it happens, there exists a nonstandard model of arithmetic admitting a computable quotient presentation by a co-c.e. equivalence relation. Actually, there are infinitely many of those. The idea of the proof consists is simulating the Henkin construction via finite injury priority argument. What is quite surprising, the construction works (i.e. injury lemma holds) by Hilbert's Basis Theorem. The latter argument is joint work with T. Slaman and L. Harrington.

Adrian Ludusan (Babeş-Bolyai University of Cluj-Napoca)

On the completeness interpretation of representation theorems

Representation theorems, similar to their counterparts, categoricity theorems, establish an isomorphism between certain algebraic systems. However, in contrast to categoricity theorems, they have received considerably little attention in the philosophy of mathematics. The presentation attempts to rectify this shortcoming by excavating the philosophical potential of representation theorems through an analysis of one of their most popular interpretations in the mathematical literature, the completeness interpretation. The meaning of this notion of completeness and the mechanism through which representation theorems are supposed to achieve it are still unclear. The paper addresses both issues. First, it proposes a definition of completeness that best suits the informal notion used in the mathematical interpretation of the theorems. Second, it formally details the mechanism responsible for achieving it. In the process, I'll issue some remarks on the significance and relevance of the formal reconstruction of the completeness interpretation for non-eliminative structuralism. For exegetical as well as evidential reasons, I'll focus on Cayley's representation theorem and use it as a case study.

Beau Madison Mount (University of Oxford)

The New Large Cardinals

In this talk, I discuss mathematical work on 'choiceless' large cardinals (plausibly consistent with ZF but known to be inconsistent with ZFC) over the last decade by Joan Bagaria, Gabriel Goldberg, Peter Koellner, Farmer Schlutzenberg, and Hugh Woodin, as well as very recent results on 'hodless' large cardinals (plausibly consistent with ZFC but known to be inconsistent with ZFC + V = HOD) by Juan Pablo Aguilera, Bagaria, Philipp Lücke, and Goldberg.

What is the philosophical significance of these results? On one view, the 'new large cardinals' demonstrate that V = HOD and choice are both restrictive principles in the same sense as V = L. I argue against this claim: there is a reasonable case for the restrictiveness of V = HOD, but it does not extend to choice. I suggest that set-theoretic realists should view theories with hodless large cardinal axioms as genuine contenders for descriptions of the universe; choiceless theories, in contrast, are ultimately to be understood instrumentally.

Antonio Piccolomini d'Aragona (University of Tübingen)

Types, sets, and models in a Kuhnian-Lakatosian perspective

I discuss the idea of a programmatic application to the history of logic of Kuhn' and Lakatos' theories for the reconstruction of the development of science. In particular, I propose a reading in Kuhnian and Lakatosian terms of the opposition betwneen realism and constructivism in logic and the foundations of mathematics. The main claim is that, in contemporary logic, one can identify a Kuhnian realist paradigm given by model theory and set theory and, next to it, a constructivist Lakatosian research programme. Although constructivism is exemplified by a number of theories not always compatible with each other, I will focus on two case-studies: Prawitz's proof-theoretic semantics and Martin-Löf's intuitionistic type theory. Finally, I will outline an epistemological framework where Kuhnian and Lakatosian ingredients can peacefully co-exist.

Cesare Straffelini (University of Barcelona)

The Inevitability of Determinacy

After defining the concept of inevitability for statements with the modal logic of forcing, we raise the question of which statements are inevitable under ZFC and under large cardinals. We show that the Π^1_1-perfect set property is ZFC-inevitable and that Projective Determinacy is inevitable assuming a proper class of Woodin cardinals. Joint work in progress with Christopher Scambler (University of Oxford).

Giorgio Venturi (University of Pisa)

Per aspera ad astra: from Skolem's Paradox to an uncountable universe

In this article, we argue in favour of the existence of uncountable collections. In particular, we contend that the universe of set theory is uncountable. Our argument is based on an analysis of Skolem's Paradox and a comparison between Cantor's Theorem and Cohen's Theorem on the existence of generic filters. We reconstruct and critically assess the skeptical argument against the notion of uncountable collections, addressing also an iterated version of this argument. Towards the end, we also connect our analysis of Skolem's Paradox to the recent discussion on Countabilism, the position that asserts everything is countable.

Xinhe Wu (London School of Economics)

Boolean-Valued Models with Urelements

We study Boolean-valued models of set theory with a proper class of urelements. We prove the fundamental theorem for Boolean-valued models with urelements concerning axiom preservation over ZFCUR. We show that certain axioms such as the $DC(\omega_1)$ scheme are preserved only by certain complete Boolean algebras. We then turn to the property of fullness. Since the standard Boolean-valued models with urelements are almost never full, we provide a different construction. The standard construction is shown to be an elementary substructure of the new construction. Finally, we prove that over ZFCUR, the Axiom of Collection is equivalent to a principle concerning the fullness of the new construction.