Intensional and extensional truth

Ionel Nariţa
West University of Timişoara

Abstract: Using the Rudolph Carnap's method of *intension* and *extension*, we can distinguish three categories of linguistic expressions: 1) *names* having a constant extension and a variable intension; 2) *terms* with a constant intension and a variable extension; 3) *descriptions* for which both intension and extension are constants. Propositions, at their turn, support two kinds of interpretations, *intensional* and *extensional*. The truth values of propositions depend on their reference as it follows: 1) the truth value of a proposition with an empty reference is given by its intensional interpretation; 2) if the reference of a proposition is not void, then the truth value of that proposition is extensionally determined. Starting from these results, some paradoxes, like "The present King of France" or *Darapti*, can be solved.

Keywords: truth value, intension, extension, paradox.

Frege has showed that, in order to explain the language phenomena, the linguistic expressions must be analyzed into two parameters named by him *Sinn* and *Bedeutung*. We'll call them here, following Carnap, *intension* and *extension*.[1] Frege started from the fact that the propositions of equality having the form "a = a" behaves differently than the propositions with the syntax "a = b". [2] For instance, let's compare the propositions:

"Hesperus = Hesperus."
"Hesperus = Phosphorus."

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[1] Carnap R., *1988*, p. 1.
[2] Frege G., *1997*, p. 563.

Both propositions are true but, while the first is necessarily true, the second is only contingently true. The coincidence between their truth value is explained by the fact that the two expressions, “Hesperus” and Phosphorus”, have the same extension and the difference concerning their modal value is due to the fact that the two expressions have different intensions.

It follows that, besides expressions there must still be another two domains, namely, the domain of extensions and the domain of intensions. The elements of the extensions’ domain are *objects* and the intensions are *senses*. In this way, according to the criteria of extension and intension, the expressions are divided into three categories:

1) Expressions that fix an extension or an object. This role is played by *names* (or *proper names*). Such expressions have the same extension in any context. For that reason, the reference of the propositions with names as their subject is immune to negation. If “a” is a name, then the propositions “*a* is *f*” and “*a* is not *f*” have the same reference. If they had a different reference then they wouldn’t be in a relation of contradiction. Because, in every context, one or another of the two propositions is true, it results that, in every context, they have the same reference, respectively, their subject, the name “a”, has the same extension in every context. We obtained the result that the extension is a constant parameter of the names; it doesn’t change in time. For example, the extension of the name “Cleopatra” was the same both in the moment when the Rosetta stone was written and in the moment when Champollion has read it.

On contrary, the intension of the names is a dynamical, variable parameter. At different moments, or in different contexts, the intension of a name has different values; the intension of a name is determined by the name’s extension and time:

$$int(a) = \varphi(ext(a), t)$$

The variable character of the names’ intension made Mill to consider that names haven’t an intension. For Mill, names have only a *denotation* and no *connotation*.[3] If we analyze names through a single parameter we can't explain why the true propositions of equality can have different modal values. Kripke tried, at his turn, to reject the intension of names.[4] He explains the difference between modal values for the propositions of equality reducing the necessity to a form of it, the necessity born from conventions. A convention remains constant in time for all users explaining the different modal values in the case of equality propositions. The two names contained in necessary equality propositions are associated with identical conventions and in the case of the contingent ones, names are connected with their extension through different conventions. However, such conventions, as *initial baptism*,[5] are just species of intension.

Consequently, we must distinguish between two kinds of names' intensions. On the one hand, names are characterized through a variable intension, which changes from a context to another and, for that reason, we'll call it *contextual intension* and, on the other hand, through a constant intension. The constant names' intension is obtained adding a temporal constant to the contextual sense. As we have seen, the contextual sense of a name is determined by name's extension and time. The name's extension being invariable, introducing a temporal constant, it results a constant intension of the name, namely, an intension as $<int(a), t_0>$ has to be constant. For example, the name "Romania" had the sense "is a kingdom" in 1938 but it hasn't this sense in present time. In change, the name "Romania" had the sense "is a kingdom in 1938" both in 1938 and now, in 2012, and it has this sense at any moment of time. In order to distinguish this invariable intension from the contextual one, we'll call it *complete intension* and its elements are *complete senses*.

2) *Terms* are the second category of expressions which have the role to fix an intension or a sense. By "term" we understand any expression used to circumscribe an intension. It follows that intension is the constant parameter of the terms. A term has the same intension in any context or at

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[3] "Proper names are not connotative: they denote the individuals who are called by them; but they do not indicate or imply any attributes as belonging to those individuals." (Mill J.S., *2009*, p. 29).

[4] Kripke S. A., *1980*, p. 33.

[5] Kripke S. A., *1980*, p. 96.

any moment. The extension of a term consists in those objects which, at a given moment, satisfy its intension. The objects satisfying a sense constitute a *class*; so, the extension of a term is a class. The extension is the dynamic parameter of the terms: the class associated to a term is changing in time. For instance, the class of the term “kingdom” contains all the objects which, at a given moment, are kingdoms. The class of kingdoms is different today, in 2012, relatively to the year 1938. While in 1938, *Romania* belonged to this class, today it isn’t an element of it. The extension of a term is determined by its intension and time:

$$ext(f) = \varphi(int(f), t)$$

The intension and extension of terms obey to the *law of inverse relation*:[6] if the intension of a term is included in the intension of a second term, then the extension of the second term is a part of the first term extension:

$$(int(f_1) \subset int(f_2)) \vdash (ext(f_2) \square ext(f_1))$$

We’ll call *null* a term with an empty class. For example, “Pegasus” is a null term, because there are no winged horses. The extension of some terms evolved in time from an empty class to a class with numerous elements. As example, the class of the term “astronaut” was empty before 12th April 1961. After this moment, it contained only one element, the Soviet astronaut Yuri Gagarin. Today, in 2012, the class of the same term has tens of elements.

3) The third category of expressions that we may distinguish using the criteria of intension and extension has the both those parameters determinate and constant. Such expressions are terms having an intension composed by complete senses. As we have seen, the complete sense of a name is constant. Therefore, an object which satisfies a complete sense at a given moment will satisfy that sense at any moment. It follows that a term

[6] Bunge M., *1974*, p. 145.

with a complete sense will have also a constant extension. We'll call this kind of terms *descriptions*.[7]

For instance, let be the terms "the river on the Eastern Romanian border" and "the river on the Eastern Romanian border in 1938". The class of the first term is variable: in 1938, for example, it contained as element the river *Dniester* and, in 2012, its element is the river *Pruth*. The class of the second term remains constant; at any moment it has a single element, the river *Dniester*. Using descriptions we manage to fix not only an intension but also an extension.

The descriptions with an elementary class as their extension are called *definite* and the others are *indefinite*. For instance, the description "Romanian King in the interwar period" is indefinite while "Romanian King in 1938" is a definite description having a class with a single element, *Carol II of Romania*.

Sometimes, the descriptions are elliptically expressed. In the example "the president of Romania" we implicitly understand that this expression refers to the *present* Romanian president. Similarly, when we say "the natural satellite of Earth" we have Moon in attention only if the present moment is implicitly understood because it is possible that, in the remote past, Earth had no natural satellites or it had more than one.

The descriptions can be used both intensionally and extensionally. For instance, in the proposition A = "The King of Romania in 1938 was a dictator", the expression "the King of Romania in 1938" is used extensionally with the purpose to determine the reference of the proposition *A*. Simultaneously, in the proposition B = "Carol II was the King of Romania in 1938", the same expression is intensionally used in order to analyze the sense of the name "Carol II".

We thus obtained the only expressions characterized by intension and extension. The names have a constant extension and a variable intension; the terms have the intension constant and the extension variable, and the description with both intension and extension constant:

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[7] The first analysis of descriptions as a distinct kind of expressions was made by Russell B., *1905*, p. 479.

	Intension	Extension
Name	dynamic parameter	constant parameter
Term	constant parameter	dynamic parameter
Description	constant parameter	constant parameter

Through a *proposition*, we *suppose* the presence of a relation concerning the parameters of the proposition's subject and predicate. The propositions containing names or terms can be understood intensionally or extensionally. Firstly, let's consider the propositions which have a name as their subject. We'll call them *elementary propositions*. Such a proposition has the form P = "*a* is *f*", where "a" is a name and "f" is a term. It has an intensional meaning:

int(f) $\subset$ int(a), "Through the proposition *P* it is supposed that the intension of the term *f* is included in the intension of the name *a*";

but also, it receives an extensional meaning:

ext(a) $\in$ ext(f), namely, "Through the proposition *P* it is supposed that the extension of the name *a* belongs to the extension of the term *f*".

The elementary propositions can be classified according to the criterion of quality in *affirmative* and *negative*. The structure of a negative elementary proposition is ~P = "*a* is not *f*". A negative proposition can be also interpreted both intensionally and extensionally as it follows:

~(int(f) $\subset$ int(a)), "Through the proposition ~P it is supposed that the intension of the term *f* is not included in the intension of the name *a*";
~(ext(a) $\in$ ext(f)), "Through the proposition ~P one supposes that the extension of the name *a* doesn't belongs to the extension of the term *f*", or equivalently, "The proposition ~P supposes that the extension of the name *a* belongs to the extension of the term ~*f*".

The subject of a *categorical proposition* is a term. In the same way as the elementary propositions, categorical propositions can be classified after their quality in *affirmative* and *negative*. Moreover, we can classify them using the criterion of *quantity* into two categories, *universal* and *particular*. The distinction between universal and particular categorical propositions concerns the *quantifiers* contained by such a proposition. Mixing these two criteria, we obtain four classes of categorical propositions, symbolized by the vowels *A*, *E*, *I* and *O*, as it follows:

Symbol	**Type**	**Structure**
A	universal affirmative	"All *f* are *g*"
E	universal negative	"No *f* is *g*"
I	particular affirmative	"Some *f* are *g*"
O	particular negative	"Some *f* are not *g*"

The intensional and extensional interpretations of categorical propositions are given in the following table:

Proposition	**Intensional interpretation**	**Extensional interpretation**
"All *f* are *g*"	int(g) □ int(f)	ext(f) □ ext(g)
"No *f* is *g*"	int(~g) □ int(f)	ext(f) □ ext(~g)
"Some *f* are *g*"	~(int(~g) □ int(f))	~(ext(f) □ ext(~g))
"Some *f* are not *g*"	~(int(g) □ int(f))	~(ext(f) □ ext(g))

For example, the proposition "All whales are mammals" can be interpreted extensionally, "The class of whales is included in the class of mammals", and intensionally, "A necessary condition to be a whale is to be a mammal". In the case of the proposition "Some birds fly", the extensional interpretation is "The class of birds is a part of the class of the flying things", and the intensional interpretation wears the form "To be a bird isn't a necessary condition for flying". We see that, the intensional interpretation represents a kind of modal interpretation of the propositions.

Since the propositions have a double interpretation, intensional and extensional, we have to assign them an *intensional truth value* and an *extensional truth value*:

A proposition is *extensionally true* if and only if its extensional interpretation *takes place*, respectively, the real relation between the subject and the predicate extensions coincides with the extensional relation supposed truugh the proposition.

A proposition is *intensionally true* if and only if its intensional interpretation *has place* in reality.

For instance, the proposition "Romania is a kingdom" is extensionally true in the situations when *Romania* belongs to the class of kingdoms and it is intensionally true when *Romania* satisfies the sense of the term "kingdom". It was both extensionally and intensionally true in the year 1938 while today, in the year 2012 it is false from both perspectives. The changing of the intensional truth of this proposition is given by the fact that the intension of the name "Romania" has changed from 1938 till 2012 and the difference between its extensional truth values is explained by the evolution of the extension of the term "kingdom" in the same period of time.

The intensional value of an elementary proposition is identical with its extensional value. If an elementary proposition is intensionally true, then it is also extensionally true and reciprocally. Let's suppose that the proposition P = "*a* is *f*" is extensionally true. In this case, the extension of the name "a" belongs to the extension of the term "f". Therefore, the object *a*, which is the extension of the name "a", being an element of the class *f*, satisfies the intension of the term "f". Consequently, the intension of the term "f" is included in the intension of "a" and the proposition *P* is intensionally true.

If we start from the hypothesis that *P* is intensionally true, then the intension of the term "f" is included in the intension of the name "a" and, in this situation, the object *a* satisfies all conditions to belong to the class of the term "f". It follows that, whether *P* is intensionally true then it is also extensionally true, namely, for the elementary propositions, the intensional and the extensional truth values are equivalent.

In change, the intensional and extensional values of the categorical propositions aren't always identical. In order to analyze the truth value of the categorical propositions it is sufficient to take into consideration the universal affirmative propositions with the syntax "All *f* are *g*". The universal negative propositions, *E*, can be brought to the *A* from performing the substitution of the predicative term with its negation, as it follows:

"No *f* is *g*" = "All *f* are ~g".

The particular categorical propositions, *O* and *I*, are the negations of the proposition *A* and *E* with the same subject and predicate. Therefore, if the truth value of the universal propositions is determined then the truth value of the particular propositions is, at its turn, determined.

As we have previously showed, the intensional and extensional interpretations of a universal affirmative proposition, "All *f* is *g*", are the following:

int(g) □ int(f)
ext(f) □ ext(g).

Concerning the relation between the intensional and the extensional truth of an *A* categorical proposition, the following statements can be proved:

1) The *A* propositions with an empty reference class haven't an extensional truth value. If ext(f) = ∅, then it takes place both ext(f) □ ext(g) and ext(f) □ ext(~g). Therefore, if the subject has an empty extension, then both *A* and *E* propositions should be extensionally true, though they are contrary. Further, if *E* is true, then its subaltern *O* shall be true, but *O* is the negation of *A*; it follows that, in such a situation, a proposition together with its negation would be true, violating the principle of contradiction. By contraposition, it results that we cannot assign an extensional truth value to

a categorical proposition with a void reference. The extensional value of such a proposition remains *undetermined*.

2) From the law of inverse relation between the intension and the extension of terms, it follows that the intensionally true propositions with a nonempty reference are also extensionally true.

3) The intensional false *A* propositions can have any extensional value. As we have seen, while the terms' intension is constant, their extension is changing in time. For such a reason, an intensionally false categorical proposition remains intensionally false no matter what is happened with the extension of its terms. Hence, between the extensions of the terms *f* and *g*, contained in an intensional false proposition, can be any relation and the extensional value of the proposition may vary from a moment to another. For instance, let be P = "All astronauts are Russians" an *A* proposition. From the intensional point of view, *P* is constantly false, because being Russian is not a necessary condition to be an astronaut at any time. On contrary, from an extensional perspective, the truth value of the proposition *P* has changed as it follows:

a) Before 12th April 1961, *P* had not an extensional determinate truth value, because no one had reached yet the cosmic space.

b) Between 12th April 1961 and 5th May 1961, the proposition *P* was extensionally true, because the only element of the class of astronauts was the Russian Yuri Gagarin.

c) After 5th May 1961, when the first American flew in the cosmic space, the proposition *P* became extensionally false.

Taking into consideration that the truth value of a proposition has to be determinate, we obtain the following principles to decide upon the *A* propositions' truth value:

1) The truth value of a categorical proposition with an empty reference is identical with its intensional truth value;

2) If the reference is not empty, then the categorical proposition's truth value is given by its extensional interpretation.

Using these principles, the truth value of a proposition can be assigned following the next table:

Extensional value	Intensional value	Proposition's truth value[8]
truth	truth	truth
truth	false	truth
false	false	false
undetermined	truth	truth
undetermined	false	false

In order to establish the truth value of a categorical proposition, C = "*kf* are/aren't *g*", we have to browse the following steps:

1) The proposition *C* is brought to the form *A* with the subject *f* and the predicate *g*. Pass to the step (2).

2) The class of the term *f* is examined.

a) If the term *f* is null, pass to (3).

b) If the term *f* is not null, pass to (4).

3) The truth value of *A* is the same with its intensional value. Pass to (5).

4) The truth value if *A* is the same with its extensional value. Pass to (5).

5) The truth value of the proposition *C* is calculated taking into account the relations from the square of opposition.

As example, let's find the truth value of the proposition I = "Some gases are poisonous":

1) The proposition *I* is brought to the form ~*A*: "~(All gases are non poisonous)";

2) The term "gases" is not null.

3) The truth value of the proposition *A* is reduced to its extensional value, being *false*.

4) Finally, the truth value of the given proposition is *truth*.

If *f* is a null term, then the proposition A = "All *f* are *g*" has to be reduced to its intensional interpretation, A = int(g) □ int(f), and, if *f* isn't a null term, then *A* is to be reduced to its extensional interpretation, namely, A = ext(f) □ ext(g).

[8] The situation when a categorical proposition is intensionally true and extensionally false is not possible.

Grounding on the intensional-extensional analysis of the propositions' truth value, several paradoxes can be solved. The paradox "The present King of France" consists in determining the truth value of the proposition K = "The present King of France is bald". B. Russell solved this paradox considering the proposition *K* false.[9] He analyzed the description "The present King of France" through the conditions of *existence*, *uniqueness*, and *adequacy*.[10] It follows that the proposition *K* can be developed into a conjunction: K1 = "There is an individual and this individual is unique and he is King of France and he is bald". Since the conjunction "There is an individual and this individual is unique and he is King of France" is false, the entire proposition K1 is also false and *K*, at its turn, is false too.

Against Russell's analysis of descriptions several objections have been raised, proving that his method to determine the truth value of the propositions with an empty reference is wrong.[11] According to Russell's analysis any proposition with a void reference should be false. M. Bunge brought as a counter-example the proposition "Zeus is the boss of the Greek Olympus"[12] about which we cannot say it is false if we accept the usually meaning of the words, though its reference is an empty class. Moreover, following the Russell's analysis we reach the absurd result that even the proposition K2 = "The present King of France is the present King of France" should be false, despite the principle of identity.[13]

If we apply the extensional-intensional analysis, the truth value of the proposition *K* is established as it follows:

1) The proposition *K* has a void reference, therefore its extensional value is undetermined and the truth value of the proposition *K* is given by its intensional interpretation.

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[9] Russell B., *1905*, p. 479.

[10] Balaiş M., *1986*, p. 13.

[11] The main reason of error in the Russell's analysis of descriptions is that the propositions like *K* must be universally quantified, and not existentially.

[12] Bunge, *1974*, p. 157.

[13] "It is false that the present King of France is the present King of France, …" (Russell B., *2000*, p. 71).

2) From an intensional perspective, *K* is a false proposition, because the condition to be bald is not necessary for being the King of France.
3) It results that the proposition *K* is false, similarly with the Russell's result.

In change, the proposition *K2* is true:
1) Since the reference class of *K2* is empty, its truth value is given by the intensional interpretation;
2) The intensional interpretation of *K2* is *true* because no one and never can be King of France without being King of France.
3) The proposition *K2* is true, despite the Russell's analysis of descriptions.

Another paradox solved through the analysis developed here is *Darapti*. In the traditional logic, the syllogistic mood AAI-3 was accepted as valid. Despite this, the corresponding formula of this mood in the logic of predicate is universally valid only if the medium term is not null. In this way, it had to recognize the existence of some syllogistic moods which are *conditionally* valid, with the consequence that syllogistics should be divided into two parts, *absolute* and *conditional*.

The *Darapti* paradox generated the thesis that particular propositions can be true only if their reference class is non empty although the universal propositions with a void reference are true. In this way, the relation of subalternation becomes conditioned. For example, according to the existential import thesis, though the universal proposition "All Martians are extraterrestrial" is true, its subaltern "Some Martians are extraterrestrial" should be false. Such a solution can't be accepted because it violates the principle of identity. Whether the particular propositions with an empty reference were all false then a proposition with the form "Some *f* are *f*" would be also false, if *f* is a null term, contrary to the identity principle.

In order to establish the validity conditions for a syllogistic mood, we have to keep into account of the intensional or extensional modality to determine the truth value of that mood's premises and conclusion. For instance, in *Darapti* case, the decision for D = (Amp & Ams) |- Isp goes as it follows:
1) If *m* is null and *s* is null, the formula *D* becomes:

D1 = ((int(p) □ int(m)) & (int(s) □ int(m))) |- ~(int(~p) □ int(s))

The formula *D1* is not valid.
2) If *m* is null and *s* isn't null then the formula *D* is interpreted through:

D2 = ((int(p) □ int(m)) & (int(s) □ int(m))) |- ~(ext(s) □ ext(~p))

The formula *D2* is not valid.
3) If *m* is not null and *s* is null, then *D* receives the interpretation:

D3 = ((ext(m) □ ext(p)) & (ext(m) □ ext(s))) |- ~(int(~p) □ int(s)) namely,
D3 = *False* |- ~(int(~p) □ int(s))

The formula *D3* is valid.
4) If both terms *m* and *s* are not nulls, the formula *D* becomes:

D4 = ((ext(m) □ ext(p)) & (ext(m) □ ext(s))) |- ~(ext(s) □ ext(~p))

The formula *D4* is valid.
We have obtained the result that the syllogisms belonging to the mood *Darapti* are correct only if their medium term is not null; hence *Darapti* is a conditioned mood. All syllogistic moods must be analyzed in the same way to establish their validity conditions. As example, let's determine the conditions of validity for the well known mood *Barbara*, B = (Amp & Asm) |- Asp:
1) If *m* is a null term and *s* is also a null term, then *B* receives the interpretation:

B1 = ((int(p) □ int(m)) & (int(m) □ int(s))) |- (int(p) □ int(s))

The formula *B1* is valid.
2) In the case that *m* is null and *s* is not null, *B* becomes:

B2 = ((int(p) □ int(m)) & (ext(s) □ ext(m))) |- (ext(s) □ ext(p))

B2 = *False* |- (ext(s) □ ext(p))

The formula *B2* is valid.
3) If *m* is not null and *m* is null, then we'll obtain:

B3 = ((ext(m) □ ext(p)) & (int(m) □ int(s))) |- (int(p) □ int(s))

The formula *B3* is not valid.
4) If both terms *m* and *s* are not nulls, then we reach the formula:

B4 = ((ext(m) □ ext(p)) & (ext(s) □ ext(m))) |- (ext(s) □ ext(p))

The formula *B4* is valid.

We see that the syllogisms of *Barbara* mood are corrects only if their medium term is null or the minor term is not null. Therefore, the validity condition for *Barbara* is that, *if the minor is null then the medium has to be null*. Despite to a widely spread opinion, *Barbara*, like all other syllogistic moods, is not absolute but conditioned.

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